Free Burnside groups of large odd exponent have cost 1

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24 August 2026

Mathematics
Münster
Cluster of Excellence

Cost

For a countable group G, let

$$\mathscr{S}(G) = \{\text{connected graphs with vertex set } G\}.$$

The **cost** of G is

$$\operatorname{cost}(G) := \inf \left\{ \frac{1}{2} \int_{\mathscr{S}(G)} \deg_{\mathscr{G}}(e_G)\, \mathrm{d}\mu(\mathscr{G}) \ : \ \begin{array}{c} \mu \text{ is a } G\text{-invariant prob.} \\ \text{measure on } \mathscr{S}(G) \end{array} \right\}.$$

- $\operatorname{cost}(\text{free group of rank } r) = r$.
- $\operatorname{cost}(\text{infinite amenable group}) = 1$.
- $\operatorname{cost}(\text{infinite group with property (T)}) = 1$ [Hutchcroft-Pete 2020].

The main result

Consider the **free Burnside group of rank *m* and exponent *n***:

$$B(m,n) := F_m / \langle\langle g^n, g \in F_m \rangle\rangle.$$

Adian 1960's: for $m \geq 2$, this group is infinite if n is odd and sufficiently large.

Theorem [Donoso-Echenique, S. '26]

For every $m \geq 2$ and every odd n sufficiently large, we have

$$\mathrm{cost}(B(m,n)) = 1.$$

Immediate consequences (due to Gaboriau's results):

- $\beta_1^{(2)}(B(m,n)) = 0.$

  extends Feldkamp-Kionke '23, who proved $\beta_1^{(2)}(B(m,p)) = 0$ with p large prime.
- $B(m,n)$ is anti-treeable.