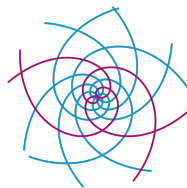


Free Burnside groups of large odd exponent have cost 1

Eduardo Silva (joint with Miguel Donoso-Echenique)

University of Münster

24 August 2026



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For a countable group G , let

$$\mathcal{S}(G) = \{\text{connected graphs with vertex set } G\}.$$

The **cost** of G is

$$\text{cost}(G) := \inf \left\{ \frac{1}{2} \int_{\mathcal{S}(G)} \deg_{\mathcal{G}}(e_G) d\mu(\mathcal{G}) : \begin{array}{l} \mu \text{ is a } G\text{-invariant prob.} \\ \text{measure on } \mathcal{S}(G) \end{array} \right\}.$$

- ▶ $\text{cost}(\text{free group of rank } r) = r.$
- ▶ $\text{cost}(\text{infinite amenable group}) = 1.$
- ▶ $\text{cost}(\text{infinite group with property (T)}) = 1$ [Hutchcroft-Pete 2020].

The main result

Consider the **free Burnside group of rank m and exponent n** :

$$B(m, n) := F_m / \langle\langle g^n, g \in F_m \rangle\rangle.$$

Adian 1960's: for $m \geq 2$, this group is **infinite** if n is odd and sufficiently large.

Theorem [Donoso-Echenique, S. '26]

For every $m \geq 2$ and every odd n sufficiently large, we have

$$\text{cost}(B(m, n)) = 1.$$

Immediate consequences (due to Gaboriau's results):

- ▶ $\beta_1^{(2)}(B(m, n)) = 0$.
extends Feldkamp-Kionke '23, who proved $\beta_1^{(2)}(B(m, p)) = 0$ with p large **prime**.
- ▶ $B(m, n)$ is anti-treeable.